Geometrical and Analytical Aspects of Observer's Mathematics

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This work considers Geometrical and Analytical aspects in a setting of arithmetic provided by Observer's Mathematics (see `www.mathrelativity.com`). We prove that Euclidean Geometry works in a sufficiently small neighborhood of the given line, but when we enlarge the neighborhood, Lobachevsky Geometry takes over. Also, we show that physical speed is a random variable and cannot exceed some constant, and this constant does not depend on an inertial coordinate system. We further consider Newton, Schrodinger, Airy equations, quantum theory of two-slit interference, wave-particle duality for single photons, and the uncertainty principle and prove some special properties for "small sizes"of nature. Certain results and communications to these theorems are also provided.

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3. *Khots, B. and Khots, D.* Quantum Theory and Observer's Mathematics, *American Institute of Physics (AIP)*, v. 965, pp 261 - 264, 2007.
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Background

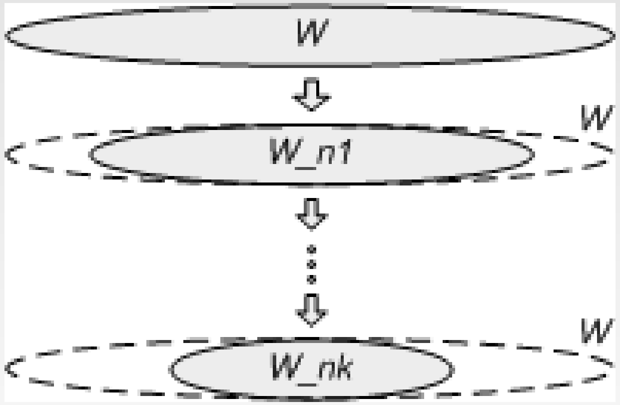
- W – set of all real numbers.
- W_n – set of all finite decimal fractions of length $2n$.
- $W_n = \{\underbrace{\star \cdots \star}_{n} . \underbrace{\star \cdots \star}_{n}\}$.
- Concept of *observers*.

Observers

- All observers are naive.
- Each *thinks* that he lives in W, but
- Each *deals* with W_n, so called W_n-observer.
- Each sees more naive observers, i.e.,
- W_{n_1}-observer can identify that W_{n_2}-observer is naive if $n_1 > n_2$.

Observers - More Specifically

- Assume $n_1 > n_2$, then
- $\star \longrightarrow \infty$ for W_{n_2}-observer means $\star \longrightarrow 10^{n_2}$ for W_{n_1}-observer.
- $\star \longrightarrow 0$ for W_{n_2}-observer means $\star \longrightarrow 10^{-n_2}$ for W_{n_1}-observer.
- For $n_1 > n_2 > \cdots > n_k$, visual example:



Arithmetic - Addition & Subtraction

- For $c = c_0.c_1 \ldots c_n$, $d = d_0.d_1 \ldots d_n \in W_n$

$$c \pm_n d = \begin{cases} c \pm d, \text{ if } c \pm d \in W_n \\ \text{not defined, if } c \pm d \notin W_n \end{cases}$$

write $\left(\left(\ldots\left(c_1 +_n c_2\right)\ldots\right) +_n c_N\right) = \sum_{i=1}^{N} {}^n c_i$ for $c_1, \ldots, c_N$ iff the contents of any parenthesis are in W_n.

Arithmetic - Multiplication

- For $c = c_0.c_1...c_n$, $d = d_0.d_1...d_n \in W_n$

$$c \times_n d = \sum_{k=0}^{n}{}^{n} \sum_{m=0}^{n-k}{}^{n} 0.\underbrace{0...0}_{k-1} c_k \cdot 0.\underbrace{0...0}_{m-1} d_m$$

where $c, d \geq 0$, $c_0 \cdot d_0 \in W_n$, $0.\underbrace{0...0}_{k-1} c_k \cdot 0.\underbrace{0...0}_{m-1} d_m$ is the standard product, and $k = m = 0$ means that $0.\underbrace{0...0}_{k-1} c_k = c_0$ and $0.\underbrace{0...0}_{m-1} d_m = d_0$. If either $c < 0$ or $d < 0$, then we compute $|c| \times_n |d|$ and define $c \times_n d = \pm |c| \times_n |d|$, where the sign $\pm$ is defined as usual. Note, if the content of at least one parentheses (in previous formula) is not in W_n, then $c \times_n d$ is not defined.

Arithmetic - Division

- Division is defined to be

$$c \div_n d = \begin{cases} r, \text{ if } \exists! r \in W_n, r \times_n d = c \\ \text{not defined, if no such } r \text{ exists or not !} \end{cases}$$

Arithmetic - General

- The arithmetic coincides with standard if the numbers are away from W_n borders.
- If the borders are *touched*, then other properties arise.
- Mathematics based on idea of observers, given these arithmetic rules:
- Observer's Mathematics – Mathematics of Relativity.
- For more info, visit `www.mathrelativity.com`.

Philosophical Aspects

- Two great Russian Geometers Rashevsky (MSU) and Norden (KSU) discussed infinite-dimensional Lie Groups.
- One of the authors was present and heard Norden's remark: "Yes, but infinity does not exist".
- Possible misuse of ordinary Differential Geometry concepts such as the limit, derivative, and integral.
- These instruments provide an advanced mathematical apparatus, with possibly faulty assumptions:
  - Space continuity
  - Functions being continuous and differentiable
- These methods and calculations may be erroneous since arbitrarily small or arbitrarily large numbers may not exist.

Geometrical Aspects

- Lines, planes, or geometrical bodies, etc exist only in our imagination.
- These shapes cannot be approached with an arbitrary accuracy due to instrument inaccuracy.
- Avoiding infinity, Hilbert had created Geometrical bases practically without the use of continuity axioms: Archimedes and completeness.
- We find similar problems occurring in Arithmetic, and in entire Mathematics, since it is "arithmetical" in nature.

Intersection of Lines

- One point:

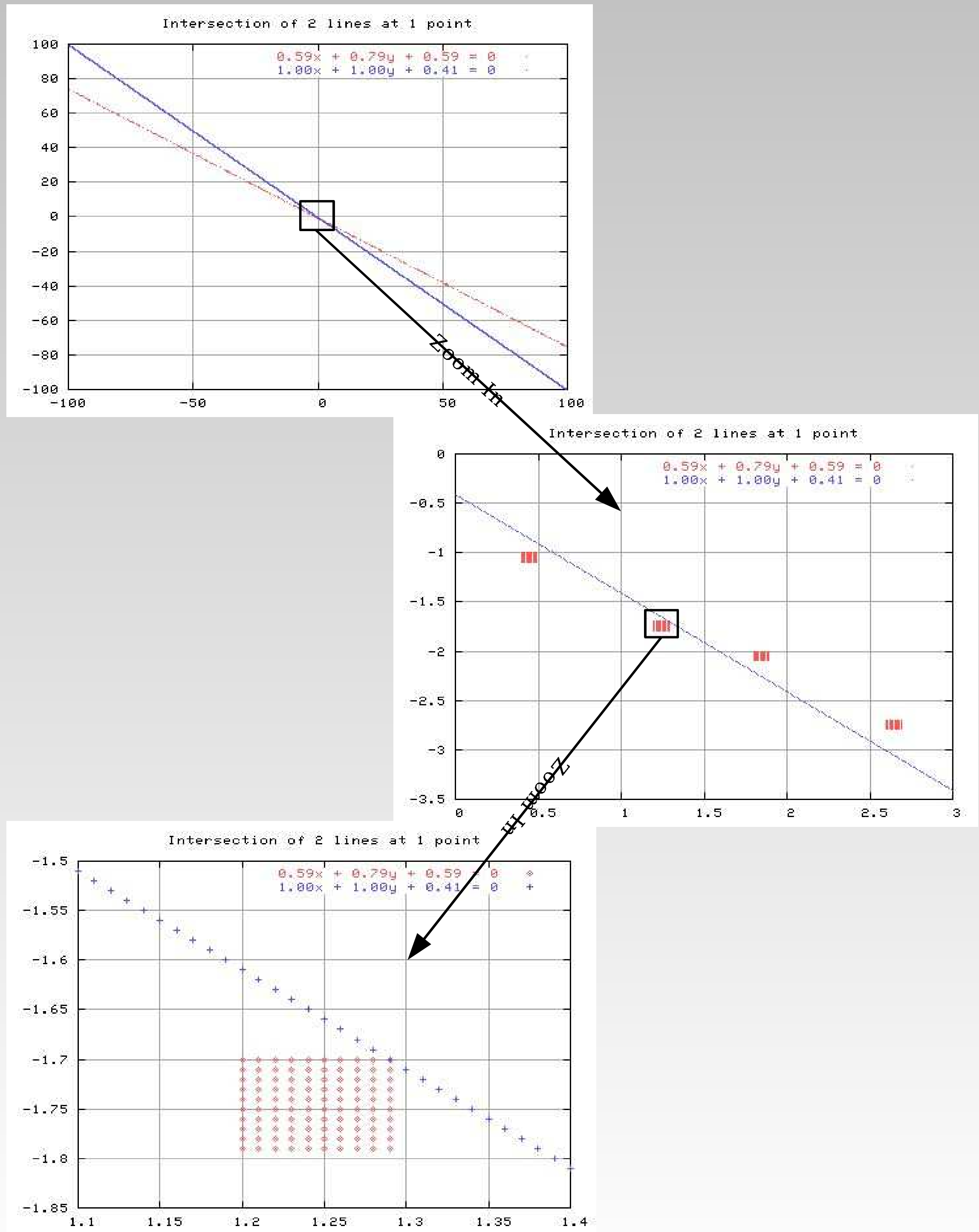
$$\begin{cases} (0.59 \times_2 x +_2 0.79 \times_2 y) +_2 0.59 = 0 \\ (1.00 \times_2 x +_2 1.00 \times_2 y) +_2 0.41 = 0 \end{cases}$$

- Two points:

$$\begin{cases} (0.31 \times_2 x +_2 1.00 \times_2 y) +_2 0.63 = 0 \\ (1.00 \times_2 x +_2 0.34 \times_2 y) +_2 0.91 = 0 \end{cases}$$

- Ten points:

$$\begin{cases} (0.08 \times_2 x +_2 0.78 \times_2 y) +_2 0.09 = 0 \\ (-0.47 \times_2 x -_2 0.75 \times_2 y) -_2 0.38 = 0 \end{cases}$$


Intersection of 2 lines at 1 point
0.59x + 0.79y + 0.59 = 0
1.00x + 1.00y + 0.41 = 0
Zoom In
Intersection of 2 lines at 1 point
0.59x + 0.79y + 0.59 = 0
1.00x + 1.00y + 0.41 = 0
Zoom In
Intersection of 2 lines at 1 point
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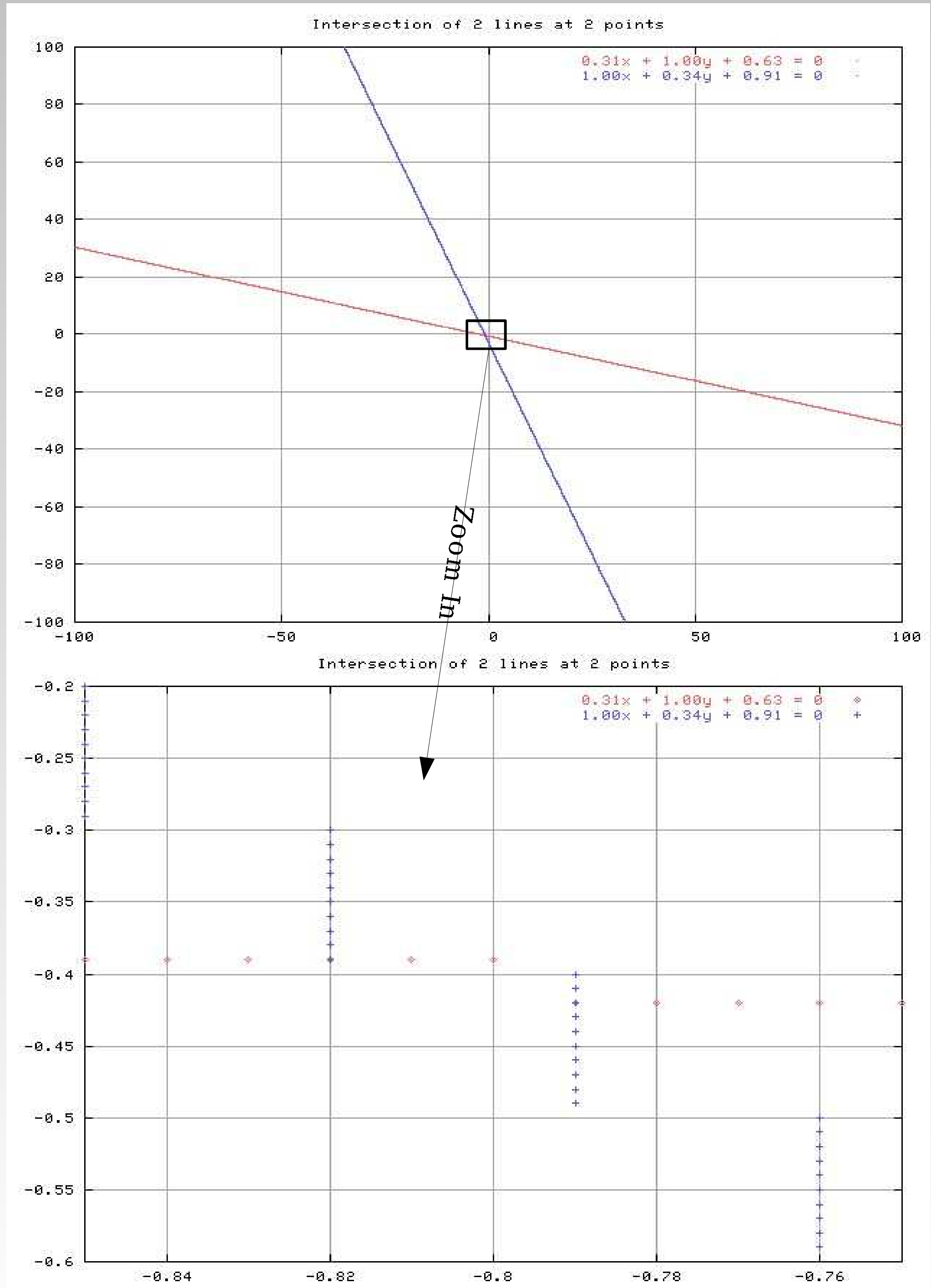
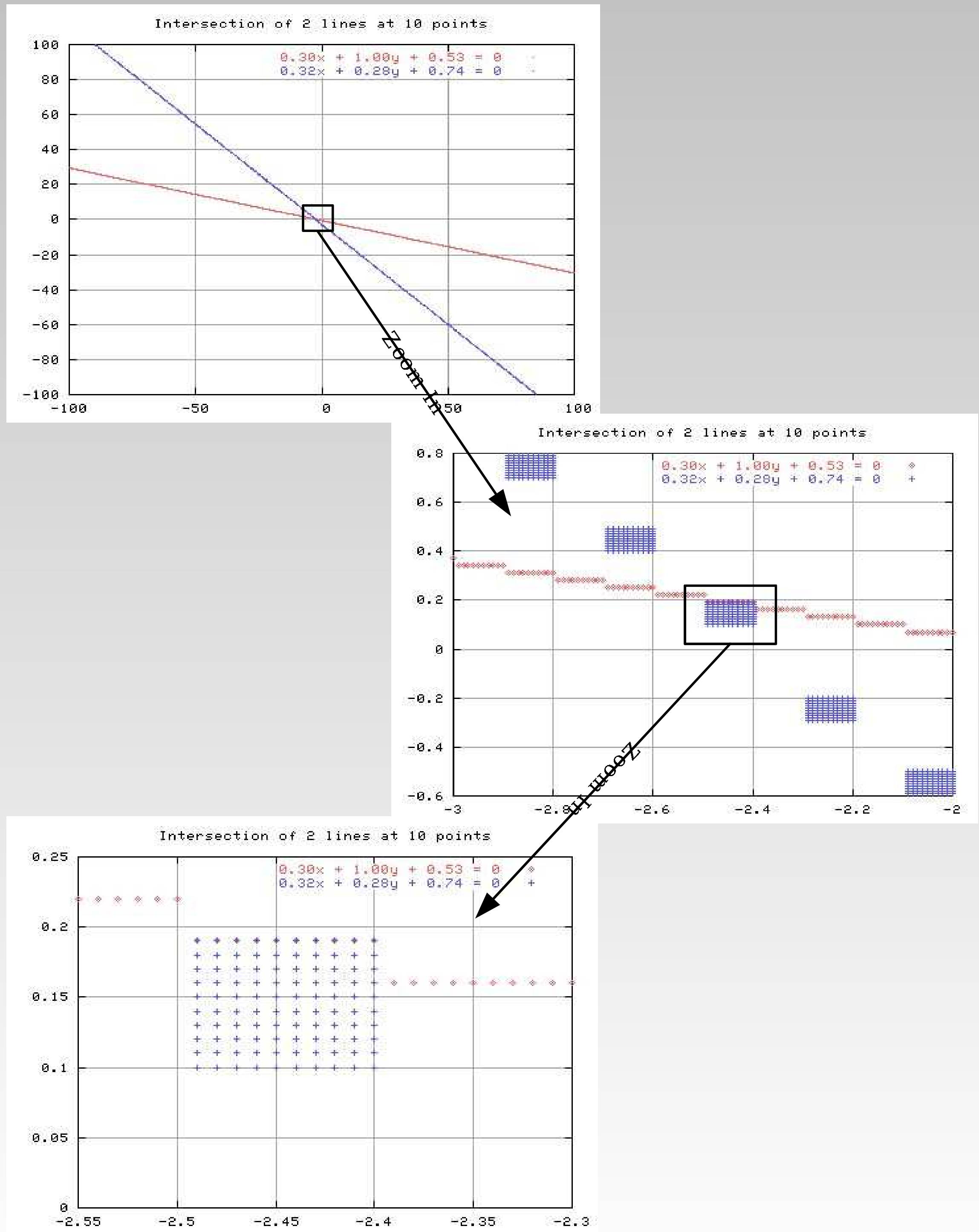

Intersection of 2 lines at 2 points
0.31x + 1.00y + 0.63 = 0
1.00x + 0.34y + 0.91 = 0
Zoom In
Intersection of 2 lines at 2 points
0.31x + 1.00y + 0.63 = 0
1.00x + 0.34y + 0.91 = 0

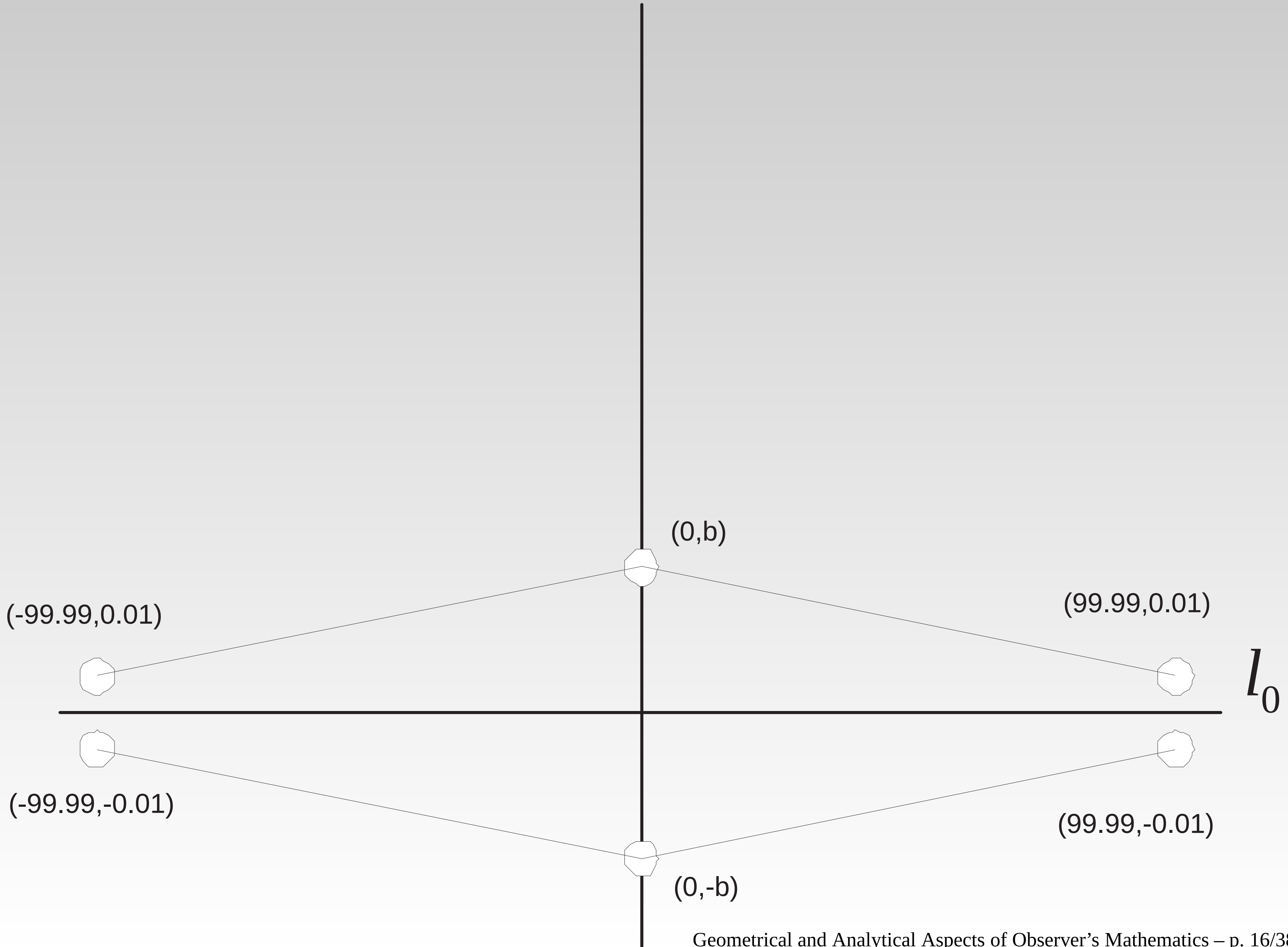

Intersection of 2 lines at 10 points
0.30x + 1.00y + 0.53 = 0
0.32x + 0.28y + 0.74 = 0
Zoom In
Intersection of 2 lines at 10 points
0.30x + 1.00y + 0.53 = 0
0.32x + 0.28y + 0.74 = 0
Zoom In
Intersection of 2 lines at 10 points
0.30x + 1.00y + 0.53 = 0
0.32x + 0.28y + 0.74 = 0

Euclidean and Lobachevsky Geometries

Theorem 1. *Given a point* $(0, b)$ *and a line* l_0*, there is a line* $y = k \times_n x +_n b$ *which is parallel to* l_0 *in Lobachevsky sense iff* $|b| \geq 1$*, and in case* $|b| < 1$*, we would only have parallel lines in Euclidean sense.*

b	k	Notes
0, 0.01,$\ldots$, 0.99	0	$\exists$ unique line through $(0, b)$ parallel to l_0 in Euclidean sense.
1, 1.01, $\ldots$, 1.98	0.01	$\exists$ lines through $(0, b)$ parallel to l_0 in Lobachevsky sense; given two values of b the lines are Euclidean parallel
1.99, 1.00, $\ldots$, 2.97	0.02	...
2.98, 2.99,$\ldots$, 3.96	0.03	...
$\ldots$	$\ldots$	...

Euclidean and Lobachevsky Geometries



Number Theory and Observer's Mathematics

- Fermat's Problem Analogy.
- **Theorem** For any n, W_n, $n \geq 2$ and for any $m \in W_n \cap \mathbb{Z}$ with $m > 2$, there exists positive $a, b, c \in W_n$, such that $a^m +_n b^m = c^m$.
- Where $x^m = (\ldots(x \times_n x) \times x \ldots)\times_n)$ for any $x \in W_n$.

Number Theory and Observer's Mathematics

- Mersenne's Problem.
- Mersenne's numbers are defined as $M_k = 2^k - 1$, with $k = 1, 2, \ldots$.
- The following question is still open: is every Mersenne's number square-free?
- **Theorem** (Analogy of Mersenne's numbers problem). There exist integers $n, k \geq 2$, Mersenne's numbers M_k, with $\{k, M_k\} \in W_n$, and positive $a \in W_n$, such that $M_k = a^2$.

Number Theory and Observer's Mathematics

- Fermat's Numbers Problem.
- Fermat's numbers are defined as $F_k = 2^{2^k} + 1$, $k = 0, 1, 2, \ldots$. T
- The following question is still open: is every Fermat's number square-free?
- **Theorem** (Analogy of Fermat's numbers problem). There exist integers $n, k \geq 2$, Fermat's numbers F_k, $\{k, F_k\} \in W_n$, and positive $a \in W_n$, such that $F_k = a^2$.

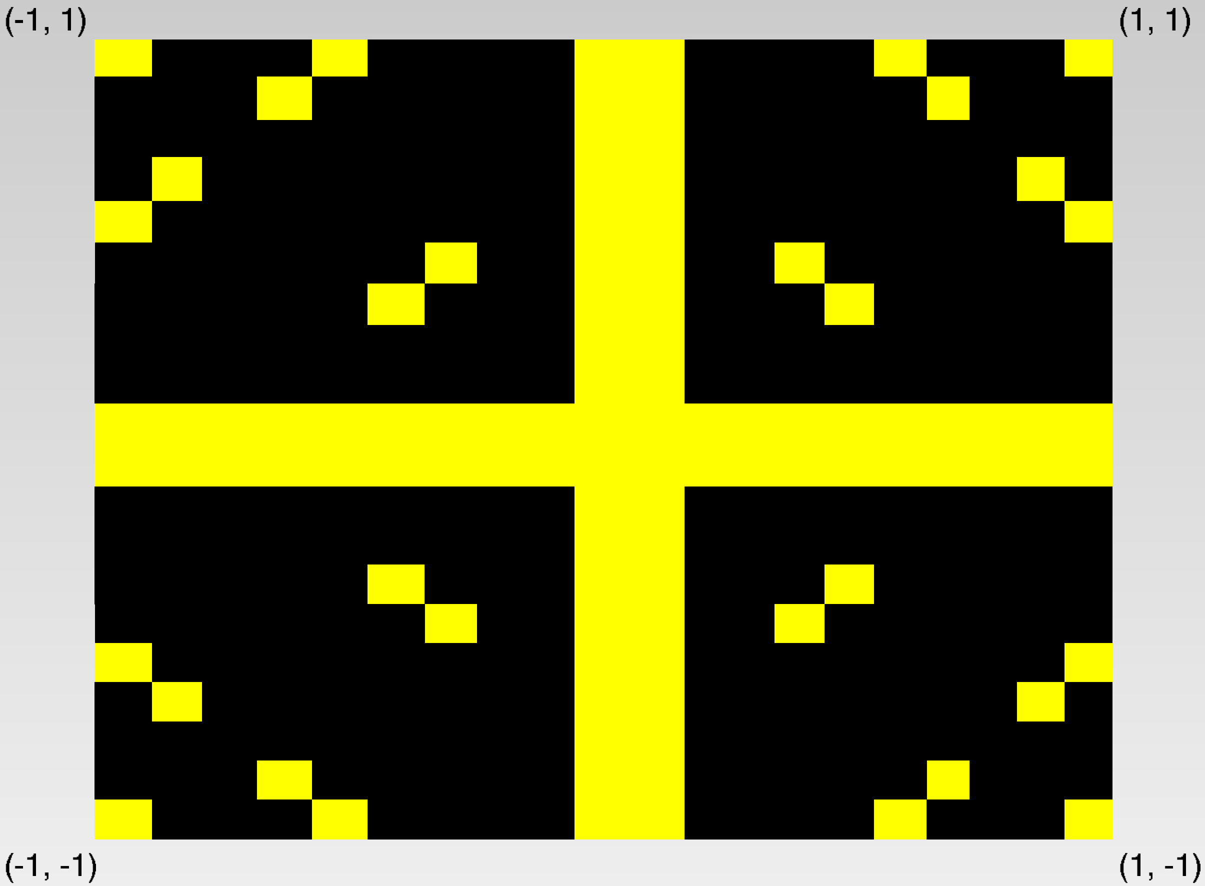
Number Theory and Observer's Mathematics

- Tenth Hilbert Problem.
- **Theorem** For any positive integers $m, n, k \in W_n$, $n \in W_m$, $m > log_{10}(1 + (2 \cdot 10^{2n} - 1)^k)$, from the point of view of the W_m-observer, there is an algorithm that takes as input a multivariable polynomial $f(x_1, \ldots, x_k)$ of degree q in W_n and outputs YES or NO according to whether there exist $a_1, \ldots, a_k \in W_n$ such that $f(a_1, \ldots, a_k) = 0$.

Observer's Math Meets Art

- Consider all segments from the origin to any point inside 2×2 square centered at origin.
- "Nadezhda" Effect: some segments do not exist, due to nonexistence of their length, given by $\sqrt{x^2 +_n y^2} = \sqrt{(x \times_n x) +_n (y \times_n y)}$

Art in W_2

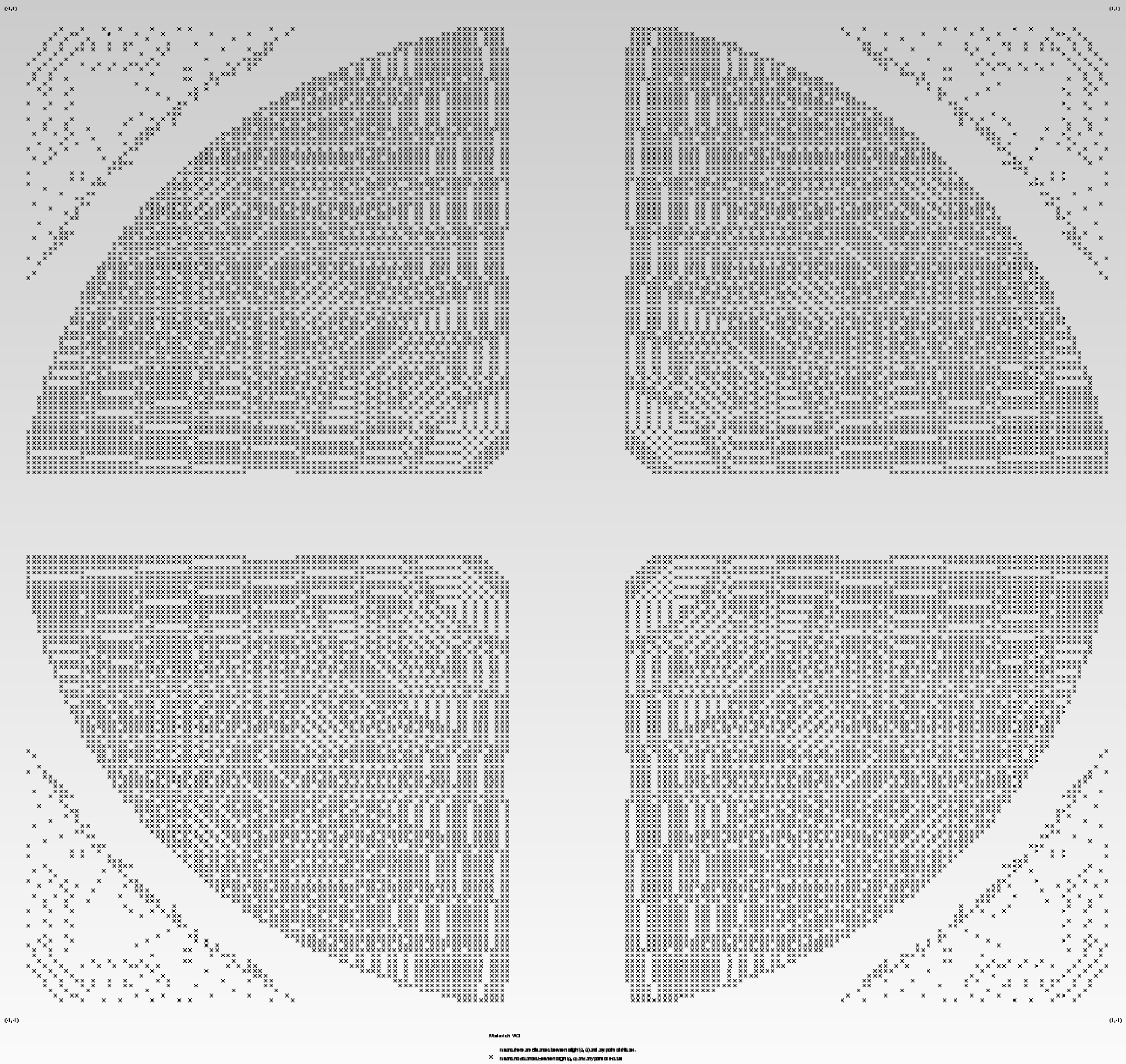


Nadezhda W2

means there are distances between origin (0, 0) and any point of this set.

means no distances between origin (0, 0) and any point of this set

Art in W_3



Physical Aspects

- Dynamics of a system change when the scale is changed at which the system is probed.
- For fluids, entire different *theories* are needed to describe behavior.
  - At ~ 1 cm - classical continuum mechanics (Navier-Stokes equations).
  - At $\sim 10^{-5}$ cm - theory of granular structures.
  - At $\sim 10^{-8}$ cm - theory of atom (nucleus + electronic cloud).
  - At $\sim 10^{-13}$ cm - nuclear physics (nucleons).
  - At $\sim 10^{-13} - 10^{-18}$ cm - quantum chromodynamics (quarks).
  - At $\sim 10^{-33}$ cm - string theory.
- Mathematical apparatus applied to math models of physical processes can operate with any numbers, which creates room for error.

Derivatives

- In W_n-observer, $y = y(x)$ is called differentiable at $x = x_0$ if there exists $y'(x_0) = \lim\limits_{x \to x_0, x \neq x_0} \frac{y(x) - y(x_0)}{x - x_0}$
- What does the above statement mean from point of view of W_m-observer with $m > n$?
- $|(y(x) -_n y(x_0)) -_n (y'(x_0) \times_n (x -_n x_0))| \leq 0.\underbrace{0 \ldots 01}_{n}$ whenever $|y(x) -_n y(x_0)| = 0.\underbrace{0 \ldots 0 y_l}_{l} y_{l+1} \ldots y_n$ and $|(x -_n x_0)| = 0.\underbrace{0 \ldots 0 x_k}_{k} x_{k+1} \ldots x_n$ for $1 \leq k, l \leq n$, and x_k - non-zero digit.

Derivatives

- **Theorem 1** From the point of view of a W_m-observer a derivative calculated by a W_n-observer ($m > n$) is not defined uniquely.

- **Theorem 2** From the point of view of a W_m-observer with $m > n$, $|y'(x_0)| \leq C_n^{l,k}$, where $C_n^{l,k} \in W_n$ is a constant defined only by n, l, k and not dependent on $y(x)$.

- **Theorem 3** From the point of view of a W_m-observer, when a W_n-observer (with $m > n \geq 3$) calculates the second derivative:

$$y''(x_0) = \lim_{x_1 \to x_0, x_1 \neq x_0, x_2 \to x_0, x_2 \neq x_0, x_3 \to x_1, x_3 \neq x_1} \frac{\frac{y(x_3)-y(x_1)}{(x_3-x_1)} - \frac{y(x_2)-y(x_0)}{x_2-x_0}}{x_1 - x_0}$$

we get the following unequality:

$$(|x_2 -_n x_0| \times_n |x_3 -_n x_1|) \times_n |x_1 -_n x_0| \geq 0.\underbrace{0\ldots01}_{n}$$

provided that $y''(x_0) \neq 0$.

Physical Interpretation

- **Hypothesis 1** Theorem 1 could offer an explanation of why physical speed (or acceleration) is not uniquely defined and, from the point of view of a measurement system (observer), it is possible to consider speed (or acceleration) as a random variable with distribution dependend on the measurement system. Let v be the speed with $v = v_0.v_1 \dots v_{n-k} + \xi_m^{n,k}$ where $\xi_m^{n,k} \in \{0.\underbrace{0\dots0}_{n-k}v_{n-k+1}\dots v_n\}$ - random variable, $m > n$, and the distribution function is $F_m^{n,k}(x) = P(\xi_m^{n,k} < x)$.
- **Hypothesis 2** Theorem 1 could offer an explanation of why the speed of any physical body cannot exceed some constant, (the speed of light, for example). Independence of this constant on explicit expression of space-time function could offer an explanation of why the speed of light does not depend on an inertial coordinate system.
- **Hypothesis 3** Theorem 2 could offer an explanation of the various uncertainty principles, when a product of a finite number of physical variables has to be not less than a certain constant. This can be seen not just from consideration of second derivatives, but of any derivative.
- **Hypothesis 4** Theorems 1, 2, and 3 combined may provide an insight into the connection between classical and quantum mechanics.

Newton Equation

- Let $F(x,t) = m \times_n \ddot{x}$. Then we have the following
- **Theorem** If the body with mass $m = m_0.m_1 \dots m_k m_{k+1} \dots m_n$, with $m \in W_n$, moves with acceleration $\ddot{x}$, $|\ddot{x}| = \ddot{x}_0.\ddot{x}_1 \dots \ddot{x}_l \ddot{x}_{l+1} \dots \ddot{x}_n$, with $\ddot{x} \in W_n$, and $m_0 = m_1 = \dots = m_k = 0$, $m_{k+1} \neq 0$, $k < n$, $\ddot{x}_0 = \ddot{x}_1 = \dots = \ddot{x}_l = 0$, $l < n$, $k + l + 2 \in W_n$, $n < k + l + 2 \leq q$, then $F(x,t) = 0$.
- **Corollary** If $l = n - 1$ and $k = 0$, i.e., $m < 1$, then $F(x,t) = 0$.
- **Theorem** If $l = n - 1$ and $\ddot{x}_n \neq 0$ then $|F(x,t)| < 9$.
- **Theorem** If $m_0 \geq \underbrace{9 \dots 9}_{p}$, $0 < p \leq n$, $\ddot{x}_0 \geq \underbrace{9 \dots 9}_{r}$, $0 < r \leq n$, $n < p + r \leq q$, then there is no force $F(x,t)$, such that $F(x,t) = m \times_n \ddot{x}$.

Schrodinger Equation

- Consider the following:
$-(\hbar \times_n \hbar) \times_n \Psi_{xx} +_n ((2 \times_n m) \times_n V) \times_n \Psi = i((2 \times_n m) \times_n \hbar)\Psi_t$, where
$\Psi = \Psi(x,t)$, $\hbar$ is the Planck's Constant, $\hbar = 1.054571628(53) \times 10^{-34}\ m^2 kg/s$.

- **Theorem** Let $36 < n < 68$, $m = m_0.m_1 \dots m_k m_{k+1} \dots m_n$, with $m \in W_n$,
$m_0 = m_1 = \dots = m_k = 0$, $m_{k+1} \neq 0$, $k + 35 < n$, $V = 0$, then
$\Psi_t = \Psi_t^0.\Psi_t^1 \dots \Psi_t^l \Psi_t^{l+1} \dots \Psi_t^n$ and $\Psi_t^0 = \dots \Psi_t^l = 0$, $\Psi_t^{l+1}, \dots, \Psi_t^n$ are free and in $\{0, 1, \dots, 9\}$, where $l = n - k - 36$, i.e., Ψ_t is a random variable, with
$$\Psi_t \in \{(0.\overbrace{\underbrace{0 \dots 0}_{l} * \dots *}^{n})\}, \text{ where } * \in \{0, 1, \dots, 9\}.$$

- **Corollary** Let $36 < n < 68$, $m = m_0.m_1 \dots m_k m_{k+1} \dots m_n$, with $m \in W_n$,
$m_0 = m_1 = \dots = m_k = 0$, $m_{k+1} \neq 0$. Also, let $V = v_0.v_1 \dots v_s v_{s+1} \dots v_n$, with
$V \in W_n$, $v_0 = v_1 = \dots = v_s = 0$, $v_{s+1} \neq 0$, with $\begin{cases} k + 35 & < & n \\ k + s + 2 & > & n \end{cases}$, then
$\Psi_t = \Psi_t^0.\Psi_t^1 \dots \Psi_t^l \Psi_t^{l+1} \dots \Psi_t^n$ and $\Psi_t^0 = \dots \Psi_t^l = 0$, $\Psi_t^{l+1}, \dots, \Psi_t^n$ are free and in $\{0, 1, \dots, 9\}$, where $l = n - k - 36$, i.e., Ψ_t is a random variable, with
$$\Psi_t \in \{(0.\overbrace{\underbrace{0 \dots 0}_{l} * \dots *}^{n})\}, \text{ where } * \in \{0, 1, \dots, 9\}.$$

Two-Slit Interference

- Let Ψ_1 wave from slit 1.
- Let Ψ_2 wave from slit 2.
- $\Psi = \Psi_1 + \Psi_2$ (with $V = 0$ in Schrodinger equation).
- **Theorem** The probability of Ψ a wave is 0.45.

Wave-Particle Duality for Single Photons

- $\lambda \times_n (m \times_n v) = h$ where h is the Planck constant.
- **Theorem** If v is small enough, then λ is a random variable.

Uncertainty Principle

- $\Delta p \times_n \Delta x = h$
- **Theorem**
  - If Δp is small enough, then Δx is a random variable.
  - If Δx is small enough, then Δp is a random variable.

Lorentz Transform

- The following is a Lorentz Transform variant in defined arithmetic
- $$\begin{cases} \lambda_1 \times_n (x -_n c \times_n t) +_n \lambda_2 \times_n (x' -_n c \times_n t') = 0 \\ \mu_1 \times_n (x +_n c \times_n t) +_n \mu_2 \times_n (x' +_n c \times_n t') = 0 \end{cases}$$
- where $|\lambda_1|$,$|\lambda_2|$,$|\mu_1|$,$|\mu_2| \geq 1$ and λ_1,λ_2,μ_1,$\mu_2 \in W_n$

Geodesic Equation

- Consider the following:

$$\ddot{x}^i +_n \sum_j {}^n \sum_k {}^n\Gamma^i_{jk} \times_n (\dot{x}^j \times_n \dot{x}^k) = 0$$

  with $j, k \in G$.

- **Theorem** If $\dot{x}^p = \dot{x}^p_0.\dot{x}^p_1 \ldots \dot{x}^p_l \dot{x}^p_{l+1} \ldots \dot{x}^p_n$, with $p \in G$, $\dot{x}^p_0 = \dot{x}^p_1 = \ldots = \dot{x}^p_l = 0$, $0 \leq l \leq n$, $n < 2l \leq q$, then we have $\ddot{x}^i = 0$, i.e., the geodesic curve is a line.

Free Wave Equation

- $u_{tt} -_n ((c \times_n c) \times_n u_{xx}) = 0$
- **Theorem 1** If c and u_{xx} are small enough, then $u_{tt} = 0$.
- **Theorem 2** If c and u_{xx} are large enough, then u_{tt} does not exist.

Airy and Korteweg-de Vries Equations

- $u_t +_n u_{xxx} = 0$
- $(u_t +_n u_{xxx}) +_n (6 \times_n (u \times_n u_x)) = 0$
- **Theorem** If u and u_x are small, then Airy equation and Korteweg-de Vries equations have the same solutions.

The Schwarzian Derivative

- $(2 \times_n (f'(x) \times_n f'(x))) \times_n S(f(x)) =$
  $2 \times_n (f'''(x) \times_n f'(x)) -_n 3 \times_n (f''(x) \times_n f''(x))$
- $S(f(x))$ is the Schwarzian Derivative.
- **Theorem** If $f'(x)$ and $f'''(x)$ are small enough and $S(f(x))$ exists, then:
  - $S(f(x))$ is a random variable.
  - $|S(f(x))| \leq 10^m$ for some $m < n$.