13. THERMODYNAMICS FROM OBSERVER'S MATHEMATICS POINT OF VIEW

13.1 Thermodynamical equations from Observer's Mathematics point of view

Thermodynamic physical quantities are those which describe macroscopic states of bodies. They include some which have both a thermodynamic and a purely mechanical significance, such as energy and volume. There are also, however, quantities of another kind, which appear as a result of purely statistical laws and have no meaning when applied to non-macroscopic systems, for example entropy.

We consider now thermal natural variables: T - temperature and S - entropy, and mechanical natural variables: P - pressure and V - volume. Also we consider thermodynamic potentials as functions of their thermal and mechanical variables: the internal energy $U = U(S, V)$ with given T and P considered as the parameters, enthalpy $H = H(S, P)$ with given T and V considered as the parameters, Helmholtz free energy $A = A(T, V)$ with given S and P considered as the parameters, and Gibbs free energy $G = G(T, P)$ with given S and V considered as the parameters.

The combined first and second thermodynamic laws give equation written in Observer's Mathematics arithmetics:

(TD1)

$$\Delta U = T \times_n \Delta S -_n P \times_n \Delta V$$

Definition of enthalpy can be written as

(TD2)

$$H = U +_n P \times_n V$$

Definition of Helmholtz free energy can be written as

(TD3)

$$A = U +_n T \times_n S$$

Definition of Gibbs free energy can be written as

(TD4)

$$G = H +_n T \times_n S$$

By **(TD1)**, **(TD2)** and **(N14)** we get

(TD2')

$$\Delta H = \Delta U +_n P \times_n \Delta V +_n V \times_n \Delta P +_n \eta_1 = T \times_n \Delta S -_n P \times_n \Delta V +_n P \times_n \Delta V +_n V \times_n \Delta P +_n \eta_1 =$$

$$= T \times_n \Delta S +_n V \times_n \Delta P +_n \eta_1$$

I.e.

$$\Delta H = T \times_n \Delta S +_n V \times_n \Delta P +_n \eta_1$$

where η_1 is a random variable depends on n and m.

We assume that all elements of this equality belong to W_n.

So, the probability of equality

$$\Delta H = T \times_n \Delta S +_n V \times_n \Delta P$$

is less than 1.

By **(TD1)**, **(TD3)** and **(N14)** we get

(TD3')

$$\Delta A = \Delta U -_n T \times_n \Delta S -_n S \times_n \Delta T +_n \eta_2 = T \times_n \Delta S -_n P \times_n \Delta V -_n T \times_n \Delta S -_n S \times_n \Delta T +_n \eta_2 =$$

$$= -P \times_n \Delta V -_n S \times_n \Delta T +_n \eta_2$$

I.e.

$$\Delta A = -P \times_n \Delta V -_n S \times_n \Delta T +_n \eta_2$$

where η_2 is a random variable depends on n and m.

We assume that all elements of this equality belong to W_n.

So, the probability of equality

$$\Delta A = -S \times_n \Delta T -_n P \times_n \Delta V$$

is less than 1.

We get

(TD4')

$$\Delta G=\Delta H -_n T\times_n \Delta S -_n S\times_n \Delta T +_n \eta_3 = T\times_n \Delta S +_n V\times_n \Delta P +_n \eta_1 -_n T\times_n \Delta S -_n S\times_n \Delta T +_n \eta_3 =$$

$$= V\times_n \Delta P +_n \eta_1 -_n S\times_n \Delta T +_n \eta_3$$

I.e.

$$\Delta G = V\times_n \Delta P -_n S\times_n \Delta T +_n \eta_4$$

where η_3, η_4 are the random variables depend on n and m.

We assume that all elements of this equality belong to W_n.

So, the probability of equality

$$\Delta G = -S\times_n \Delta T +_n V\times_n \Delta P$$

is less than 1.

13.2 Maxwell relations in Thermodynamics from Observer's Mathematics point of view

We get

$$T = \partial U/\partial S +_n \eta_5$$

and

$$-P = \partial U/\partial V +_n \eta_6$$

where η_5, η_6 are random variables depend on n and m.

And we get

(OMTD1)

$$\partial T/\partial V = -\partial P/\partial S +_n \eta_7$$

where η_7 is a random variable depends on n and m.

We assume that all elements of **(OMTD1)** belong to W_n.

We get

$$T = \partial H/\partial S +_n \eta_8$$

and

$$V = \partial H/\partial P +_n \eta_9$$

where η_8, η_9 are random variables depend on n and m.

And we get

(OMTD2)

$$\partial T/\partial P = \partial V/\partial S +_n \eta_{10}$$

where η_{10} is a random variable depends on n and m.

We assume that all elements of **(OMTD2)** belong to W_n.

By **(N16)** and **(TD3')** we get

$$-S = \partial A/\partial T +_n \eta_{11}$$

and

$$-P = \partial A/\partial V +_n \eta_{12}$$

where η_{11}, η_{12} are random variables depend on n and m.

And we get

(OMTD3)

$$\partial S/\partial V = \partial P/\partial T +_n \eta_{13}$$

where η_{13} is a random variable depends on n and m.

We assume that all elements of **(OMTD3)** belong to W_n.

We get

$$-S = \partial G/\partial T +_n \eta_{14}$$

and

$$V = \partial G/\partial P +_n \eta_{15}$$

where η_{14}, η_{15} are random variables depend on n and m.

And we get

(OMTD4)

$$-\partial S/\partial P = \partial V/\partial T +_n \eta_{16}$$

where η_{16} is a random variable depends on n and m.

We assume that all elements of **(OMTD4)** belong to W_n.

THEOREM 13.1. *Equations* ***(OMTD1)***, ***(OMTD2)***, ***(OMTD3)*** *and* ***(OMTD4)*** *are Maxwell relations in Thermodynamics from Observer's Mathematics point of view.*