5. SPECIAL RELATIVITY FROM OBSERVER'S MATHEMATICS POINT OF VIEW

5.1 Zero-divisors, non-associativity and non-distributivity, Lorentz transformation in Observer's Mathematics

Let us consider the Observer's Mathematics point of view. We consider all events below as appurtenant to W_n for some n, and point of view belongs to W_m with $m > n$. Here we do not take numerical estimation of m, but for us it is enough that W_m observer can see all sets of numbers which we operate on each step. A light-signal, which is proceeding along the positive axis of x, is transmitted according to the equation

$$x = c \times_n t$$

or

$$x -_n c \times_n t = 0$$

Since the same light-signal has to be transmitted relative to K' with the velocity c, the propagation relative to the system K' will be represented by the analogous formula $x' -_n c \times_n t' = 0$. At that the disappearance of $(x -_n c \times_n t)$ involves the disappearance of $(x' -_n c \times_n t')$, and vice versa. If we apply quite similar considerations to light rays which are being transmitted along the negative x-axis, we obtain the analogous condition:

$$x +_n c \times_n t = 0$$

and

$$x' +_n c \times_n t' = 0$$

And also at that the disappearance of $(x +_n c \times_n t)$ involves the disappearance of $(x' +_n c \times_n t')$, and vice versa. We would like to say that this will be the case when the relation

$$x' -_n c \times_n t' = \lambda \times_n (x -_n c \times_n t)$$

is fulfilled in general, where λ indicates a constant, $\lambda \neq 0$. Also we would like to say that this will be the case when the relation

$$x' +_n c \times_n t' = \mu \times_n (x +_n c \times_n t)$$

is fulfilled in general, where μ indicates a constant, $\mu \neq 0$.

The critical aspect is that all of these statements are wrong in Observer's Mathematics, because Observer's Mathematics has zero-divisors. For example, if we take $n = 2$, $\lambda = 0.8$ and $x -_n c \times_n t = 0.08$ then $x' -_n c \times_n t' = \lambda \times_n (x -_n c \times_n t) = 0$. Same situation takes a place with μ. Thus, if we have $|\lambda| < 1$, then the statement "the case when the relation $x' -_n c \times_n t' = \lambda \times_n (x -_n c \times_n t)$ is fulfilled in general" becomes wrong. And also if we have $|\mu| < 1$, statement "the case when the relation $x' +_n c \times_n t' = \mu \times_n (x +_n c \times_n t)$ is fulfilled in general" becomes wrong. So, relations **??** and **??** above become wrong from Observer's Mathematics point of view. But in case $\lambda \geq 1, \mu \geq 1$ both statements are correct. We proved above that in classical case we have relation $\lambda\mu = 1$ (it is classical multiplication here). It means that if $\lambda > 1$ (and in reality $\lambda > 1$), then $\mu < 1$. We can see analogous situation in Observer's Mathematics case. So, we have to change classical approach and write down the first principal of Special Theory of Relativity using the following equalities:

$$\begin{cases} x' -_n c \times_n t' = \lambda \times_n (x -_n c \times_n t) \\ \mu \times_n (x' +_n c \times_n t') = x +_n c \times_n t \end{cases} \tag{1}$$

where $\lambda \geq 1$, $\mu \geq 1$, and λ, μ are constants. For the origin of K' we have permanently $x' = 0$, and $x = v \times_n t$. It means:

$$\begin{cases} -c \times_n t' = \lambda \times_n (v \times_n t -_n c \times_n t) \\ \mu \times_n (c \times_n t') = v \times_n t +_n c \times_n t \end{cases} \tag{2}$$

From here we have

$$\mu \times_n (-\lambda \times_n (v \times_n t -_n c \times_n t)) = v \times_n t +_n c \times_n t$$

for all t.

Now we are going back to the equation:

$$\mu \times_n (\lambda \times_n (c \times_n t -_n v \times_n t)) = c \times_n t +_n v \times_n t$$

for all t. With probability P_1, $0 < P_1 < 1$ we can rewrite this equation as:

$$\mu \times_n (\lambda \times_n ((c -_n v) \times_n t)) = (c +_n v) \times_n t$$

With probability P_2, $0 < P_2 < 1$ we can rewrite this equation as:

$$\mu \times_n ((\lambda \times_n (c -_n v)) \times_n t) = (c +_n v) \times_n t$$

With probability P_3, $0 < P_3 < 1$ we can rewrite this equation as:

$$(\mu \times_n (\lambda \times_n (c -_n v)) \times_n t = (c +_n v) \times_n t$$

With probability P_4, $0 < P_4 < 1$ we can rewrite this equation as:

$$((\mu \times_n (\lambda \times_n (c -_n v)) -_n (c +_n v)) \times_n t = 0$$

Because this equation has to be fulfilled for all t, we can rewrite this equation as:

$$\mu \times_n (\lambda \times_n (c -_n v)) = c +_n v$$

THEOREM 5.1. *The statement: "Equation*

$$\mu \times_n (\lambda \times_n (c -_n v)) = c +_n v$$

is equivalent to equation

$$\mu \times_n (\lambda \times_n (c \times_n t -_n v \times_n t)) = c \times_n t +_n v \times_n t$$

for all t" has probability more than 0 and less than 1.

THEOREM 5.2. *The statement "Equation* $(\mu \times_n \lambda) \times_n (c -_n v) = c +_n v$ *is equivalent to equation* $\mu \times_n (\lambda \times_n (c \times_n t -_n v \times_n t)) = c \times_n t +_n v \times_n t$ *for all* t*" has probability more than* 0 *and less than* 1.

So, relation above may be correct only with probability which is more than 0 but less than 1, from Observer's Mathematics point of view. Furthermore, the second principle of relativity states that, as judged from K, the length of a unit measuring-rod which is at rest with reference to K' must be exactly the same as the length, as judged from K', of a unit measuring-rod which is at rest relative to K. In order to see how the points of the x'-axis appear as viewed from K, we only require to take a "snapshot" of K' from K; this means that we have to insert a particular value of t (time of K), e.g. $t = 0$. So, $c \times_n t' = x' -_n \lambda \times_n x$ and $\mu \times_n (2 \times_n x' -_n \lambda \times_n x) = x$. Let's take $x'_0 = 0$ and $x'_1 = 1$ and then find corresponding x_0 and x_1. Then we get the following:

$$\mu \times_n (-\lambda \times_n x_0) = x_0$$

With probability P_5 , $0 < P_5 < 1$, we can rewrite this equation as:

$$(\mu \times_n \lambda) \times_n x_0 +_n x_0 = 0$$

With probability P_6 , $0 < P_6 < 1$, we can rewrite this equation as:

$$((\mu \times_n \lambda) +_n 1) \times_n x_0 = 0$$

If $\mu \times_n \lambda > 0$, $x_0 = 0$. So, we proved that $x_0 = 0$ with probability P_7, $0 < P_7 < 1$. x_1 is a solution of equation:

$$\mu \times_n (2 -_n \lambda \times_n x_1) = x_1$$

But if the snapshot would be taken from K' ($t' = 0$), we obtain from the second set of the equations

$$\begin{cases} x' = \lambda \times_n (x -_n c \times_n t) \\ \mu \times_n x' = x +_n c \times_n t \end{cases} \tag{3}$$

So, $c \times_n t = \mu \times_n x' -_n x$ and $x' = \lambda \times_n (2 \times_n x -_n \mu \times_n x')$ Let's take $x_3 = 0$, $x_4 = 1$, and then find corresponding x'_3 and x'_4.

$$x'_3 = \lambda \times_n (-\mu \times_n x'_3)$$

With probability P_8 , $0 < P_8 < 1$, we can rewrite this equation as:

$$x'_3 +_n (\lambda \times_n \mu) \times_n x'_3 = 0$$

With probability P_9 , $0 < P_9 < 1$, we can rewrite this equation as:

$$(1 +_n (\lambda \times_n \mu)) \times_n x'_3 = 0$$

If $\mu \times_n \lambda > 0$, $x'_3 = 0$. So, we proved that $x'_3 = 0$ with probability P_{10}, $0 < P_{10} < 1$. x'_4 is a solution of equation: $x'_4 = \lambda \times_n (2 -_n \mu \times_n x'_4)$ But from what has been said, the two snapshots must be identical; hence x_1 must be equal to x'_4, so that we obtain:

$$x_1 = x'_4$$

So, relations above may be correct only with probability which is more than 0 but less than 1, from Observer's Mathematics point of view. And finally we have a system of equations:

$$\begin{cases} \mu \times_n (\lambda \times_n (c -_n v)) = c +_n v \\ \mu \times_n (2 -_n \lambda \times_n x_1) = x_1 \\ x'_4 = \lambda \times_n (2 -_n \mu \times_n x'_4) \\ x_1 = x'_4 \end{cases} \tag{4}$$

Now we denote $x_1 = x'_4 = x_2$. And we can rewrite this system as a system with 3 equations:

$$\begin{cases} \mu \times_n (\lambda \times_n (c -_n v)) = c +_n v \\ \mu \times_n (2 -_n \lambda \times_n x_2) = x_2 \\ \lambda \times_n (2 -_n \mu \times_n x_2) = x_2 \\ 0 < v < c \\ \lambda \geq 1 \\ \mu \geq 1 \end{cases} \tag{5}$$

So, general transformation given by equations with λ and μ satisfying a system of equations is a 2-dimensional analog of classical Lorentz transformation. We will call this transformation Observer's Mathematics Lorentz transformation. So, we proved

Theorem 5.3. *Observer's Mathematics Lorentz transformation is satisfying to the first and second principals of Special Theory of Relativity with probability P, $0 < P < 1$.* Note we can state the same if we substitute equation $\mu \times_n (\lambda \times_n (c -_n v)) = c +_n v$ for $(\mu \times_n \lambda) \times_n (c -_n v) = c +_n v$.

5.2 Observer's Mathematics Lorentz Transformation Characteristics

Let's consider the system of equations defining constants λ and μ (with given v) of Observer's Mathematics Lorentz transformation.

Theorem 5.4. *The constants λ and μ are both > 1 automatically, i.e., the system of equations defining constants λ and μ may be written downs as follows:*

$$\begin{cases} \mu \times_n (\lambda \times_n (c -_n v)) = c +_n v \\ \mu \times_n (2 -_n \lambda \times_n x_2) = x_2 \\ \lambda \times_n (2 -_n \mu \times_n x_2) = x_2 \\ 0 < v < c \end{cases} \tag{6}$$

Let's now consider solutions to the existing question. For any given v, $0 < v < c$, solutions are the sets $\{\mu, x_2\}$. Let's consider for example $n = 2$, i.e., $x, t, x', t', c, v, \lambda, \mu, x_2 \in W_2$. Put $c = 1$, then $0 < v < 1$, i.e., $v \in \{0.01, 0.02, \ldots, 0.98, 0.99\}$. Also, let's assume $\lambda = \mu$. The full set of solutions is presented in the table below.

v	$\lambda = \mu$	x_2
0.16	1.2; 1.21; 1.22	0.99
0.2	1.23; 1.24; 1.25; 1.26; 1.27	0.99
0.21	1.28; 1.29	0.99
	1.3	0.97
0.28	1.36; 1.37; 1.38; 1.39	0.97
0.56	1.9; 1.91; 1.92; 1.93; 1.94	0.82
0.57	1.95; 1.96; 1.97; 1.98; 1.99	0.82
0.6	2; 2.01; 2.02	0.8
0.74	2.64; 2.65	0.68
0.75	2.66; 2.67; 2.68	0.68
0.76	2.73; 2.74; 2.75	0.66
0.77	2.8; 2.81	0.64
0.78	2.87; 2.88; 2.89	0.64
0.8	3; 3.01	0.6
0.85	3.55	0.53
0.96	7; 7.01; 7.02; 7.03; 7.04; 7.05; 7.06; 7.07	0.28

First of all, solution λ does not exist for each v. Moreover, for some of v solution λ is not unique. And for the found pair $\{v, \lambda\}$ solution x_2 does not always exist. Thus, we can state the following.

THEOREM 5.5. *Probability of the existance of Observer's Mathematics Lorentz transformation with given v, $0 < v < c$, in W_n is less than* 1.

Let's consider now classical Lorentz transformation effects such as time delay, relativity of simultaneity, and length contraction from point of view Observer's Mathematics Lorentz transformation. Let's start from length contraction. Let's take $n = 2$, $x'_s = 0$, $x'_f = 1$, $t_s = t_f = 0$, $c = 1$, $v = 0.57$, and $\lambda = \mu = 1.95$, where the index s denotes segment origin, and index f means segment end. Put this data into the system 1 to get the following system.

$$\begin{cases} 0 -_2 t'_s = \lambda \times_2 (x_s -_2 0) \\ \mu \times_2 (0 +_2 t'_s) = x_s +_2 0 \end{cases} \tag{7}$$

and $x_s = 0$

$$\begin{cases} 1 -_2 t'_f = \lambda \times_2 (x_f -_2 0) \\ \mu \times_2 (1 +_2 t'_f) = x_f +_2 0 \end{cases} \tag{8}$$

and $x_f = 0.82$. So, we have $x_f -_2 x_s = 0.82 < x'_f -_2 x'_s = 1$, i.e., in this case we have length contraction. Let's now take $n = 2$, $x'_s = 0$, $x'_f = 0.01$, $t_s = t_f = 0$, $c = 1$, $v = 0.57$, and $\lambda = \mu = 1.95$. We get again $x_s = 0$ and the following system.

$$\begin{cases} 0.01 -_2 t'_f = \lambda \times_2 (x_f -_2 0) \\ \mu \times_2 (0.01 +_2 t'_f) = x_f +_2 0 \end{cases} \tag{9}$$

and $x_f = 0.01$. So, we have $x_f -_2 x_s = 0.01 = x'_f -_2 x'_s$, i.e., in this case, there is not length contraction. And finally let's take $n = 2$, $x'_s = 0$, $x'_f = 2.14$, $t_s = t_f = 0$, $c = 1$, $v = 0.57$, and $\lambda = \mu = 1.95$. We get again $x_s = 0$ and the following system.

$$\begin{cases} 2.14 -_2 t'_f = \lambda \times_2 (x_f -_2 0) \\ \mu \times_2 (2.14 +_2 t'_f) = x_f +_2 0 \end{cases} \tag{10}$$

and x_f does not exist. So, we can state

THEOREM 5.6. *In Observer's Mathematics Lorentz transformation, the length of segment* $[x_s, x_f]$ *in coordinate system* K *may:*

- *be less than its length in coordinate system* K',
- *be equal to its length in coordinate system* K',
- *not exist.*

Let's now consider the relativity of simultaneity effect. Let's take $n = 2$, $x_a = 0$, $x_b = 1$, $t_a = t_b = 0$, $c = 1$, $v = 0.57$, and $\lambda = \mu = 1.95$. Put this data into the system 1 to get the following system.

$$\begin{cases} x'_a -_2 t'_a = 0 \\ \mu \times_2 (x'_a +_2 t'_a) = 0 \end{cases} \tag{11}$$

and $t'_a = 0$.

$$\begin{cases} x'_b -_2 t'_b = \lambda \times_2 (x_b) \\ \mu \times_2 (x'_b +_2 t'_b) = x_b \end{cases} \tag{12}$$

and $t'_b = -0.7$. So, $t'_b -_2 t'_a = -0.7 \neq 0$, i.e., in this case, we have the relativity of simultaneity effect. Let's now take $n = 2$, $x_a = 0$, $x_b = 0.01$, $t_a = t_b = 0$, $c = 1$, $v = 0.57$, and $\lambda = \mu = 1.95$. We have in this case $t'_a = 0$ and $t'_b = 0$, i.e., in this case, we do not have the relativity of simultaneity effect. And finally letý take $n = 2$, $x_a = 0$, $x_b = 0.48$, $t_a = t_b = 0$, $c = 1$, $v = 0.57$, and $\lambda = \mu = 1.95$. We get again $t'_a = 0$ and t'_b does not exist. So, we can state the following.

THEOREM 5.7. *In Observer's Mathematics Lorentz transformation simultaneous events in coordinate system* K *may:*

- *not be simultaneous in coordinate system* K',
- *be simultaneous in coordinate system* K',
- *not exist.*

Let's now consider the time delay effect. Let's take $n = 2$, $x'_a = x'_b = 0$, $t_a = 0$, $t_b = 1$, $c = 1$, $v = 0.57$, and $\lambda = \mu = 1.95$. We can calculate: $t'_a = 0$, $t'_b = 0.82$ and $t'_b -_2 t'_a = 0.82 < t_b -_2 t_a = 1$, i.e., in this case, there is time delay. Let's now take $n = 2$, $x'_a = x'_b = 0$, $t_a = 0$, $t_b = 0.01$, $c = 1$, $v = 0.57$, and $\lambda = \mu = 1.95$. We have in this case $t'_a = 0$, $t'_b = 0.01$ and $t'_b -_2 t'_a = 0.01 = t_b -_2 t_a$, i.e., in this case, there is no time delay. And finally let's take $n = 2$, $x'_a = x'_b = 0$, $t_a = 0$,

$t_b = 0.48$, $c = 1$, $v = 0.57$, and $\lambda = \mu = 1.95$. We have $t'_a = 0$, t'_b does not exist. So, we can state the following.

THEOREM 5.8. *In Observer's Mathematics Lorentz transformation interval of time on clocks in coordinate system K' may:*

- *be less than interval of time on clocks in coordinate system K,*
- *equal to interval of time on clocks in coordinate system K,*
- *not exist.*